Damping mechanisms of the Δ resonance in nuclei

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The damping mechanisms of the $\Delta(1232)$ resonance in nuclei are studied by analyzing the quasi-free decay reactions ^{12}C(π^+,π^+p)^{11}B and ^{12}C(^{3}He,tπ^+p)^{11}B and the 2p emission reactions ^{12}C(π^+,pp)^{10}B and ^{12}C(^{3}He,tpp)^{10}B. The coincidence cross sections are calculated within the framework of the isobar-hole model. It is found that the 2p emission process induced by the decay of the Δ resonance in the nucleus can be consistently described by a $\pi+\rho+g'$ model for the Δ+N$\rightarrow$ N+N decay interaction.

I. INTRODUCTION

Recently, new information on the $\Delta(1232)$-propagation in the nucleus has been obtained from a coherent pion decay experiment [1,2] and from a $(\vec{p},\vec{n})$ spin-flip transfer experiment [3,4]. In the first experiment the ^{12}C(^{3}He,tπ^+)^{12}C(g.s.) reaction was used to measure the isovector spin-longitudinal $(\vec{S}\cdot\vec{q}\,\vec{T})$ response function in the Δ resonance region. In the second experiment the spin observables of the $(\vec{p},\vec{n})$ reaction were used to decompose the charge exchange cross section in the Δ resonance region into its spin-longitudinal (LO) and spin-transverse (TR) components. Similar to the π-nucleus total cross section data [5,6,7,8] the LO cross sections of both reactions show a substantial downward energy shift of the Δ resonance in nuclei, as compared to the proton target. From a consistent Δ-hole model analysis of pion and photon scattering, and charge exchange reactions [8,9,10,11] it is found that a large part of the observed shift is due to a nuclear medium effect on the LO response function. The medium effect is caused by the strongly attractive, energy dependent Δ-particle - nucleon-hole residual interaction $V_{\Delta N,\Delta N}$. In Refs. [8,9,10,11,12] it was shown that $V_{\Delta N,\Delta N}$ can be well described by the $\pi+\rho+g'$ model ([13] and references therein). The strong attraction of the π-exchange in the LO channel produces a collective pion mode at excitation energies of ~ 250 MeV in the laboratory frame. The collectivity shifts the LO response function down in energy by 60 MeV relative to the spin-transverse $(\vec{S}\times\vec{q}\,\vec{T})$ response function. Other, smaller effects come from Δ conversion processes, such as $\Delta N \rightarrow N+N$ [9,10], and from projectile excitation [14,15].

In the present paper we apply the Δ-hole model of Refs. [8,11] to the calculation of the damping of the collective pion mode in the nucleus. The major decay channels are the coherent pion decay, the quasi-free decay, and the 2p emission. While various calculations for the coherent pion decay were published already in Refs. [8,11,12,16,17] we give here the results for the quasi-free (π^+p) decay and the 2p emission. Since the coupling interaction for the quasi-free decay is known we can use this process to study the distortion effects on the outgoing pion and proton wave functions. For the 2p emission process we assume a $\pi+\rho+g'$ interaction. We show that the 2p emission in the Δ resonance energy region is dominated by the zero-range Landau-Migdal interaction, the strength of which can be exctracted from the data. Both the pion induced reactions and the charge exchange reactions are well reproduced by calculations with a Landau-Migdal parameter in the range of $g'_{N\Delta}\approx 0.25-0.35$.

II. THEORY

In this section we describe the Δ-hole model used in the analysis of the experimental data. The formalism and the methods of calculation were presented already in recent papers [8,11]. In the present paper we discuss only those formulas which are connected with the quasi-free decay and the 2p emission.

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A. The excitation processes

We start our formulation by writing down the fivefold differential cross section for the charge exchange reaction $A + a \to (B + \Delta) + b \to C + c + d + b$ in the LAB frame ($E_A = m_A$, $\vec{p}_A = 0$)

$$\frac{d^5\sigma}{dE_b d\Omega_b dE_c d\Omega_c d\Omega_d} = \frac{1}{(2\pi)^8} \frac{1}{\mid \vec{v}_{rel} \mid} \frac{m_a}{E_a} \frac{m_b}{E_b} \frac{m_c}{E_c} \frac{m_d}{E_d} \frac{m_C}{E_C} \frac{p_b\, E_b\, p_c\, E_c\, p_d\, E_d\, E_C}{\mid (m_A + \omega_{c.e.} - E_c) - \frac{E_d}{p_d}(p_a \cos\theta_{ad} - p_b \cos\theta_{bd} - p_c \cos\theta_{cd}) \mid}$$
$$\times \overline{\sum} \mid T_{fi} \mid^2 \quad . \tag{1}$$

Here A $(B + \Delta)$ and a (b) denote the target (excited intermediate nucleus) and projectile (ejectile), respectively. The $B + \Delta$ system de-excites to the residual nucleus $\mid \varphi_C \rangle$ by emission of the particles c and d which carry four-momenta $(E_c, \vec{p}_c)$ and $(E_d, \vec{p}_d)$, respectively. m_i stands for the mass of particle i $(i = A, C, a, b, c, d)$ and $(\omega_{c.e.}, \vec{q}_{c.e.})$ denotes the four-momentum transfer in the excitation process. In case that one of the outgoing particles is a boson (e.g. in the quasi-free decay) the according normalisation factor M/E has to be replaced by 1/2E. The full four body kinematics in the final reaction channel is included.

The transition amplitude T_{fi} for the decay process is defined as

$$T_{fi} = \langle \varphi_C, [\phi_c(\vec{p}_c)\ \phi_d(\vec{p}_d)]_{(\mathcal{A})} \mid V_{cd,\Delta} \mid \psi \rangle \tag{2}$$

where $V_{cd,\Delta}$ denotes the Δ decay interaction that will be specified later. $\phi_c(\vec{p}_c)$ and $\phi_d(\vec{p}_d)$ are the distorted wave functions of the outgoing particles c and d, respectively, and φ_C is the wave function of the residual nucleus. The index $(\mathcal{A})$ indicates that in case of the 2p emission the wave functions of the two outgoing identical fermions have to be antisymmetrized. The wave function $\mid \psi \rangle$ describes the intermediate $B + \Delta$ system and is defined by [11]

$$\mid \psi \rangle = G \mid \rho \rangle = \frac{1}{\omega + i\Gamma_\Delta/2 - H_B - T_\Delta - U_\Delta - V_{\Delta N, \Delta N}} \mid \rho \rangle \tag{3}$$

where $\mid \rho \rangle$ is the doorway state excited initially by the reaction. The Green's function G describes the propagation of the $(B + \Delta)$ system and is approximated by that of the isobar-hole model [18,19,20,21]. $\Gamma_\Delta(\omega)$ is the energy dependent free decay width of the Δ, H_B is the Hamiltonian of the hole nucleus B, T_Δ and U_Δ are the kinetic energy operator and the Δ-nucleus one-body potential, respectively, and $V_{\Delta N, \Delta N}$ is the Δ-hole residual interaction. For the calculation of $|\psi\rangle$ we use the same input parameters as used in Refs. [8,11]. The Pauli blocking effects are assumed to be included in the average Δ-nucleus one-body potential, which we fixed by re-analysing the relevant scattering data [5,6]. We refer the reader for more details to Refs. [8,11].

For charge exchange reactions the doorway state $\mid \rho \rangle$ has the explicit form [11]

$$\mid \rho_{c.e.} \rangle = (\chi_b^{(-)} \varphi_b \mid t_{NN,N\Delta} \mid \chi_a^{(+)} \varphi_a \varphi_A \rangle \tag{4}$$

where $\chi_a^{(+)}$ and $\chi_b^{(-)*}$ denote the projectile and ejectile distorted wave functions, respectively, and φ_a and φ_b are the corresponding intrinsic wave functions of the projectile a and ejectile b; $\mid \varphi_A \rangle$ describes the target ground state wave function. The effective $NN \to N\Delta$ transition operator for the charge exchange process is denoted by $t_{NN,N\Delta}$. The round bra $(\ \mid$ on the right side of eq. (4) denotes the integration with respect to the projectile coordinates only.

For pion induced reactions the coincidence cross section in the LAB frame is threefold differential

$$\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_d} = \frac{1}{(2\pi)^5} \frac{1}{\mid \vec{v}_{rel} \mid} \frac{1}{2\omega_\pi} \frac{m_c}{E_c} \frac{m_d}{E_d} \frac{m_C}{E_C} \frac{p_c E_c p_d E_d E_C}{\mid (m_A + \omega_\pi - E_c) - \frac{E_d}{p_d}(p_a \cos\theta_{ad} - p_c \cos\theta_{cd}) \mid} \overline{\sum} \mid T_{fi} \mid^2 \tag{5}$$

where $(\omega_\pi, \vec{q}_\pi)$ is the four-momentum of the incident pion. Now the doorway state has the form [8]

$$\mid \rho_\pi \rangle = \frac{f_{\pi N\Delta}}{m_\pi} \left(\vec{q}_\pi \cdot \vec{S}^\dagger \right)\ T_\nu^\dagger\ e^{i\vec{q}_\pi \cdot \vec{r}} \mid \varphi_A \rangle \tag{6}$$

where $\vec{S}^\dagger$ and $\vec{T}^\dagger$ are the spin and isospin transition operators, respectively, that convert a nucleon into a $\Delta(1232)$ isobar. The coupling constant $f_{\pi N\Delta}$ is fixed from pion-nucleon scattering data and has the value $f_{\pi N\Delta}^2/4\pi = 0.324$. The index $\nu = \pm 1$ distinguishes between $\pi^\pm$ scattering.

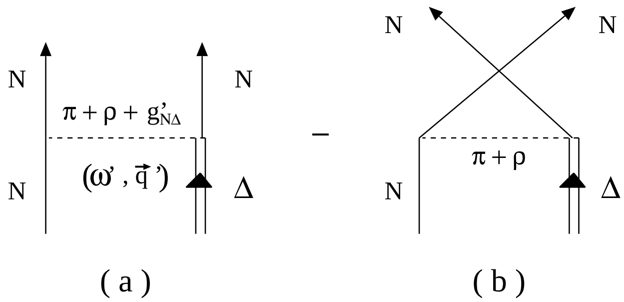

FIG. 1. Schematic representation of the 2p emission induced by the decay of the Δ: (a) direct graph, (b) exchange graph.

B. The decay interactions

In case of the ^{12}C(π^+,π^+p)^{11}B and ^{12}C(^{3}He,tπ^+p)^{11}B reactions the decay interaction $V_{cd,\Delta}$ of eq. (2) is represented by

$$V_{p\pi,\Delta} \;=\; \frac{f_{\pi N\Delta}}{m_\pi}\, \vec{S}\cdot\vec{q}^{\,\prime}_{\pi}\, T_\mu . \tag{7}$$

Note that this interaction has no free parameter and is known from elastic pion scattering in the Δ resonance region. The explicit formulas for the quasi-free decay of the Δ are given in appendix A.

In case of the ^{12}C(π^+,pp)^{10}B and ^{12}C(^{3}He,tpp)^{10}B reactions the interaction for the process $\Delta + N \to N + N$ is described by a $\pi + \rho + g'$ model [13]:

$$V_{pp,\Delta}(\omega',\vec{q}^{\,\prime}) \;=\; V_\pi(\omega',\vec{q}^{\,\prime}) \;+\; V_\rho(\omega',\vec{q}^{\,\prime}) \;+\; V_\delta \;\; , \tag{8}$$

with

$$V_\delta \;=\; \hbar c\, \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2}\, g'_{N\Delta}\, \left(\vec{\sigma}_2\cdot\vec{S}_1\right)\left(\vec{\tau}_2\cdot\vec{T}_1\right) \;\; . \tag{9}$$

In eq. (8), ω' and $\vec{q}^{\,\prime}$ are the energy and three-momentum transfer involved in the interaction $\Delta + N \to N + N$, respectively. The interaction V_δ is the so-called Landau-Migdal term. It describes the short range correlations for $\Delta + N \to N + N$ transitions. The special value for the Landau-Migdal parameter $g'_{N\Delta} = 1/3$ (in units of $\hbar c f_{\pi NN} f_{\pi N\Delta}/m_\pi^2 \approx 800$ MeV fm^3) is known as the 'minimal $g'_{N\Delta}$' because it cancels out the attractive short range part of the π-echange potential [13].

The π- and ρ-exchange potentials V_π and V_ρ are defined consistently with the potentials for the residual interaction:

$$V_\pi(\omega',\vec{q}^{\,\prime}) = \hbar c\, \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t'}\right)^2 \frac{1}{t' - m_\pi^2 + i\varepsilon}\, (\vec{\sigma}_2\cdot\vec{q}^{\,\prime})(\vec{S}_1\cdot\vec{q}^{\,\prime})\, \left(\vec{\tau}_2\cdot\vec{T}_1\right) \tag{10}$$

$$V_\rho(\omega',\vec{q}^{\,\prime}) = \hbar c\, \frac{f_{\rho NN} f_{\rho N\Delta}}{m_\rho^2} \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - t'}\right)^2 \frac{1}{t' - m_\rho^2 + i\varepsilon}\, (\vec{\sigma}_2\times\vec{q}^{\,\prime})\cdot(\vec{S}_1\times\vec{q}^{\,\prime})\, \left(\vec{\tau}_2\cdot\vec{T}_1\right) \;. \tag{11}$$

In the eqs. (10) and (11), $t' = \omega'^2 - \vec{q}^{\,\prime 2}$ is the four-momentum transfer in the decay process, m_π and Λ_π (m_ρ, Λ_ρ) are the mass and cutoff mass of the π (ρ), respectively. The various parameters are fixed as follows: $f_{\pi NN} f_{\pi N\Delta}/4\pi = 0.162$, $f_{\rho NN} f_{\rho N\Delta}/4\pi = 8.32$, $m_\pi = 0.14$ GeV, $m_\rho = 0.77$ GeV, $\Lambda_\pi = 1.20$ GeV, and $\Lambda_\rho = 2$ GeV.

As a consequence of the Pauli principle the wave functions of the two outgoing protons have to be antisymmetrized. This leads to two contributions to the 2p emission process, namely the direct and the exchange term (see Fig. 1). The antisymmetrization has to be carried out only for the finite-range π- and ρ-exchange potentials. The Landau-Migdal term is a zero-range interaction and has not to be antisymmetrized. This treatment of the $\Delta + N \to N + N$ interaction is in line with microscopic nuclear structure calculations [22,23] and microscopic G-Matrix calculations [24]. Therefore the Landau-Migdal parameter $g'_{N\Delta}$ extracted from the 2p emission reactions can be directly compared with the values found in these calculations [22,23,24]. The explicit formulas for the 2p emission process induced by the decay of the Δ in the nucleus are derived in appendix B.

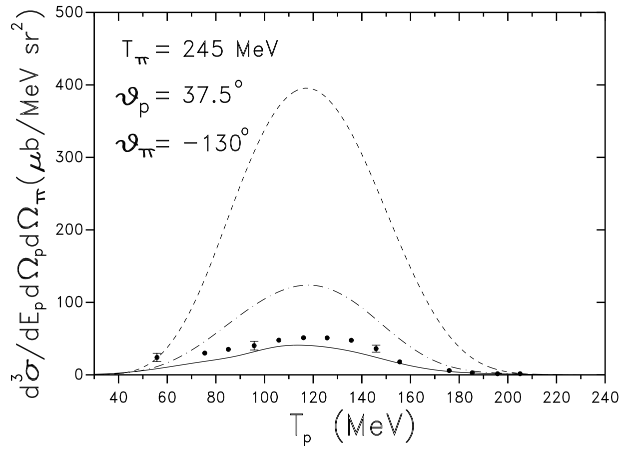

FIG. 2. Triple differential cross section of the ^{12}C(π^+,pπ^+)^{11}B reaction at given pion and proton angle plotted versus the kinetic energy of the decay proton. The data have been taken from Ref. [27]. The dashed curve shows the result without inclusion of the residual interaction $V_{\Delta N,\Delta N}$ while the dot-dashed curve shows the result with the inclusion of $V_{\Delta N,\Delta N}$. The solid curve shows the result with additional inclusion of the distortion effect on the decay pion and proton wave functions.

III. RESULTS AND DISCUSSION

With the formalism described in Sec. II we have calculated cross sections for the quasi-free decay reactions ^{12}C(π^+,π^+p)^{11}B and ^{12}C(^{3}He,tπ^+p)^{11}B and the 2p emission reactions ^{12}C(π^+,π^+pp)^{10}B and ^{12}C(^{3}He,tπ^+pp)^{10}B. While the pion induced reactions are truely exclusive with the decay particles measured in coincidence at given energies and angles the decay cross sections of the charge exchange reaction have been integrated over a certain kinematical range, as determined by the geometry of the DIOGENE detector [1,2]. Therefore the integrated cross sections for (^{3}He,t) induced processes are in a way less exclusive than the pion induced reactions and show only the gross features of the process.

Furthermore, our calculations treat the distorsion effects on the incoming and outgoing particles in the adequate frameworks. In the pion induced reaction the distorsion of the incoming pion is treated within the isobar hole model while the distorsion of the decay pion and protons is described by optical model wave functions [25,26]. In the (^{3}He,t) charge exchange reaction the projectile and ejectile and the decay particles are described by optical model wave functions, whereas the Δ propagation through the nucleus is again treated within the isobar-hole model.

A. The reaction ^{12}C(π^+,pπ^+)^{11}B

In Fig. 2 we compare the results of our calculations for the ^{12}C(π^+,pπ^+)^{11}B reaction to the data of Ref. [27] at the pion kinetic energy of $T_\pi = 245$ MeV. In the experiment the angles of the outgoing proton and pion were fixed at $\theta_p = 37.5°$ and $\theta_\pi = -130°$, respectively, in coplanar geometry. The threefold differential cross section is plotted versus the kinetic energy of the outgoing proton. Three different calculations are compared to the data. The dashed curve shows the result of our calculations with $V_{\Delta N,\Delta N} = 0$ while the dot-dashed curve shows the result with $V_{\Delta N,\Delta N} \neq 0$. By comparison of both curves one recognizes that the inclusion of the residual interaction reduces the quasi-free decay cross section by a factor of ~ 4. This reduction is due to the absorption taking place in the multiple scattering of the pion. A similar reduction factor is also observed in the total pion-nucleus cross section [8]. The solid curve shows the result with additional inclusion of the distortion effect on the outgoing proton and pion wave functions. The distortion effect leads to a further reduction of the cross section by a factor of ~ 3. We describe the relative motion of the decay particles with respect to the residual nucleus by optical model wave functions. This is a consistent method within the framework of direct nuclear reaction theory and has been used in the analysis of other reactions, like A(e,e' pp). For the calculation of the proton and pion wave functions we used the optical potential parameters, as derived from elastic proton-nucleus [25] and elastic pion-nucleus scattering [26]. Using these optical model wave functions we overestimate the absorption and thus obtain a lower limit for the ^{12}C(π^+,pπ^+)^{11}B cross section. A comparison of the solid curve with the data shows that we underestimate the data by $\sim 10-20\%$. This is in agreement with our expectation and assures us that our treatment of the distortion effects is reasonable. In the following calculations we will use the same model for the description of the distortion effects on the outgoing particles.

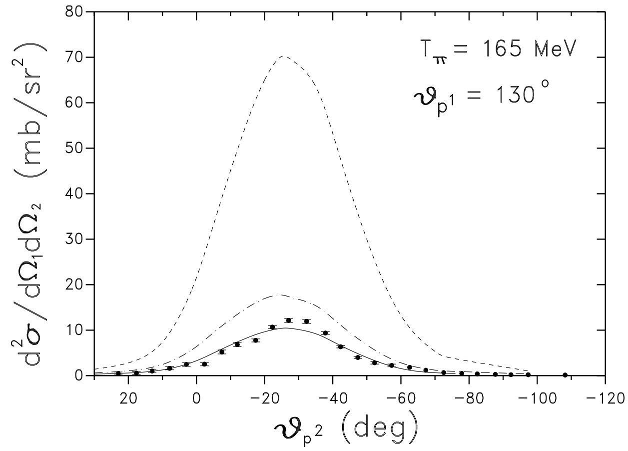

FIG. 3. Coincidence spectra for the ^{12}C(^{3}He,t) reaction at $T_{He} = 2$ GeV and triton scattering angle $\theta_t = 0°$. (a) The ^{12}C(^{3}He,tpπ^+) data [2] in comparison with the result of our calculation. (b) The ^{12}C(^{3}He,tpp) reaction data [2]. The solid and dashed curve show the results with and without inclusion of the residual interaction, respectively.

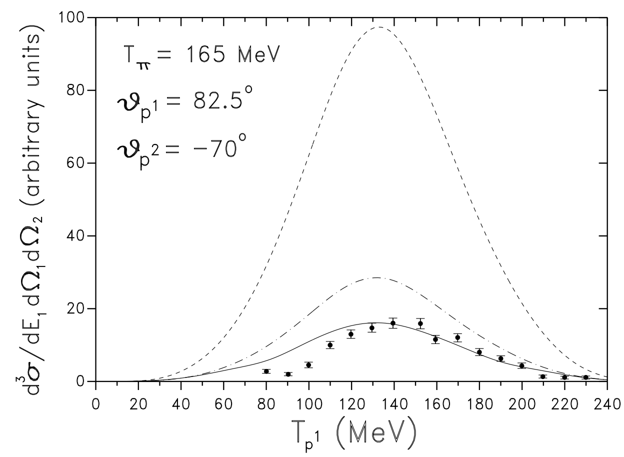

FIG. 4. Double differential cross section of the ^{12}C(π^+,pp)^{10}B reaction. The angle of proton 1 is fixed at $\theta_{p^1} = 130°$. The cross section is plotted versus the angle of proton 2. The data have been taken from Ref. [28,29]. The dashed curve shows the result with $V_{\Delta N,\Delta N} = 0$ while the dot-dashed curve shows the result with $V_{\Delta N,\Delta N} \neq 0$. The solid curve shows the result with additional inclusion of the distortion of the decay protons.

B. The reaction ^{12}C(π^+,pp)^{10}B

In Fig. 3 we show the results for the ^{12}C(π^+,pp)^{10}B cross section at $T_\pi = 165$ MeV. The experiment was performed in coplanar geometry with the angle of proton 1 fixed at $\theta_{p^1} = 130°$. The threefold differential cross section is plotted versus the angle of proton 2. The dashed curve shows the result of our calculation with $V_{\Delta N,\Delta N} = 0$ while the dashed-dotted curve shows the result with $V_{\Delta N,\Delta N} \neq 0$. The inclusion of the multiple scattering of the pion in the medium results again in a reduction of the cross section by a factor of ~ 4. The solid curve shows the result with additional inclusion of the distortion effect on the outgoing proton wave functions. This leads to a further reduction of the cross section by a factor of ~ 2. The calculations shown here were performed with a minimal Landau-Migdal parameter $g'_{N\Delta} = 1/3$ in the 2p emission matrix element, which gives a very good agreement with the data [28]. Neglection of the π- and ρ-exchange potentials in the decay interaction leads to almost the same results. Thus the Landau-Migdal term is the most important ingredient to the $\Delta + N \rightarrow N + N$ interaction. This is a consequence of the large momentum and energy transfer involved in the $\Delta + N \rightarrow N + N$ decay process leading to a very short ranged interaction. We remark that the determination of the $g'_{N\Delta}$ parameter in the (π^+,pp)-reaction is rather direct since this reaction is dominated by the intermediate Δ^{++} excitation. On the other hand, in case of the ^{12}C(π^+,pn)^{10}C reaction [29] or photon induced

2p emission reactions [30,31] many competing processes are possible and the Landau-Migdal parameter $g'_{N\Delta}$ can only be extracted in an indirect way.

In Fig. 4 we study the same reaction as in Fig. 3; here the two protons were detected at $\theta_{p^1} = 82.5°$ and $\theta_{p^2} = -70°$, and the cross section is plotted as function of the kinetic energy of proton 1. The curves shown have the same meaning as in Fig. 3. Note that the data are given in arbitrary units. Thus the data are normalized to the solid curve. The inclusion of the residual interaction and the inclusion of the distortion effects reduces the magnitude again by a factor of ~ 4 and ~ 2, respectively. The dependence of the cross section on the kinetic energy of the proton 1 is reproduced well. The slight deviation of the calculated cross section from the data at low kinetic energies could indicate that the treatment of the distortion on the outgoing nucleons should be improved for these energies.

C. The reaction ^{12}C(^{3}He,tpπ^+)

In Fig. 5 (a) we compare our results for the ^{12}C(^{3}He,tpπ^+) reaction with the data of Hennino *et al.* [2]. The data are integrated over the kinematically allowed pion and proton energies as well as pion and proton angles. The solid curve shows our calculation with inclusion of the residual interaction $V_{\Delta N,\Delta N}$ and with inclusion of the distortion effects on the outgoing particles. We see no effect of the residual interaction in this energy and angle integrated cross section, i. e. the cross sections with and without inclusion of $V_{\Delta N,\Delta N}$ are the same. This is an indication that the pπ^+ events in this reaction come only from the nuclear surface.

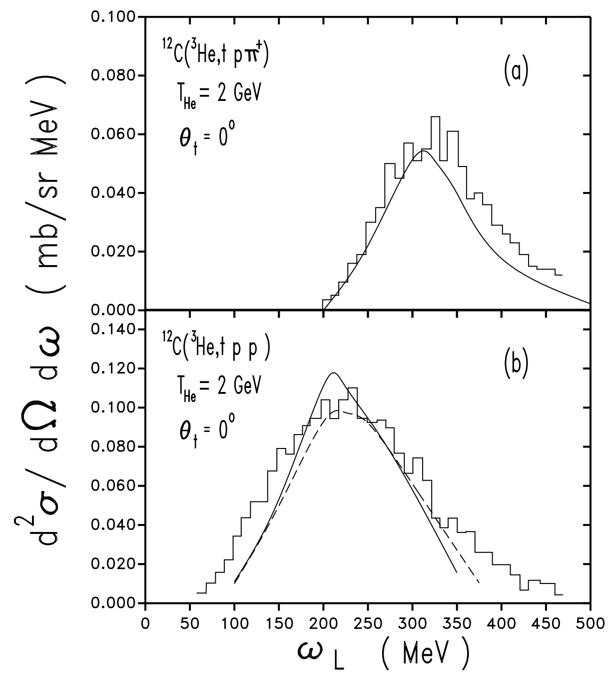

FIG. 5. Threefold differential cross section of the ^{12}C(π^+,pp)^{10}B reaction. The two protons were detected at $\theta_{p^1} = 82.5°$ and $\theta_{p^2} = -70°$, respectively. The cross section is plotted as function of the kinetic energy of proton 1. The data have been taken from Ref. [28,29]. The curves shown have the same meaning as in Fig. 3. Note that the data are given in arbitrary units and are thus normalized to the solid curve.

D. The reaction ^{12}C(^{3}He,tpp)

In Fig. 5 (b) we compare our microscopically calculated ^{12}C(^{3}He,tpp) coincidence spectra with the data of Hennino *et al.* [2]. The data are integrated over the phase space of both outgoing protons. Two calculations are compared to the data: The solid curve shows the result with $V_{\Delta N,\Delta N} \neq 0$ while the dashed curve shows the result with $V_{\Delta N,\Delta N} = 0$. In both cases the distortion effects are taken into account. In order to reproduce the magnitude of the experimental data we used a Landau-Migdal parameter of $g'_{N\Delta} = 0.28$. This value is consistent with that found in the (π^+,pp)-reactions of Figs. 3 and 4. Both values for $g'_{N\Delta}$ lie in the range from $0.25 - 0.35$ and are thus in agreement with values of the Landau-Migdal parameter found in microscopic G-Matrix calculations [24].

IV. SUMMARY

In conclusion, we have presented microscopic calculations for the quasi-free decay of the Δ resonance and the 2p emission in nuclei induced by pion absorption and by charge exchange reactions. These calculations are performed within the framework of the Δ-hole model and are consistent with our former calculations of inclusive and exclusive reactions on nuclei. Since the coupling interaction for the quasi-free decay is known we use this process to study the distortion effects on the outgoing pion and proton wave functions. In the 2p emission reactions we describe the Δ+N$\rightarrow$ N+N decay interaction by a $\pi + \rho + g'$ model, which is consistent with our description of the residual interaction. We find that the $\Delta + N \rightarrow N + N$ decay interaction is dominated by the zero-range Landau-Migdal term. The data for both the (π^+,pp) reaction and the (^{3}He,tpp) reaction are well reproduced by calculations with a Landau-Migdal parameter in the range of $g'_{N\Delta} \approx 0.25 - 0.35$.

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APPENDIX A: EXPLICIT FORMULAS FOR THE QUASI-FREE DECAY

In this appendix we derive the explicit formulas for the transition amplitude of the quasi-free decay. Inserting the decay interaction $V_{p\pi,\Delta}$ of eq. (7) into eq. (2) one can write the transition amplitude in the following way:

$$T_{q.f.} = \langle J_h^{'} M_h^{'}; \vec{q}_p^{\,\prime} \frac{1}{2}\, m_s'; \vec{q}_\pi^{\,\prime} \mid \frac{f_{\pi N\Delta}}{m_\pi}\, \vec{S}\cdot\vec{q}_\pi^{\,\prime}\, T_\mu \mid \psi\rangle \ . \tag{A1}$$

Here $J_h^{'}$ and $M_h^{'}$ represent the total angular momentum and the magnetic quantum number of the final (A-1)-nucleus; $\vec{q}_p^{\,\prime}$ and m_s' are the three-momentum and the spin projection of the outgoing proton; $\vec{q}_\pi^{\,\prime}$ is the three-momentum of the outgoing pion. We expand the wave function $\mid\psi\rangle$ of the Δ-hole state in terms of the channel wave functions [11]

$$\mid [Y_\Delta \Phi_h]_{j_t m_t}\rangle = \sum_{m_\Delta m_h} (j_\Delta\, m_\Delta\, j_h\, m_h \,|\, j_t\, m_t)\ \mid Y_{j_\Delta m_\Delta} \Phi_{j_h m_h}\rangle \ , \tag{A2}$$

where $Y_{j_\Delta m_\Delta}$ is the spin-angle wave function of the Δ and $\Phi_{j_h m_h}$ is the hole wave function of nucleus B. Thus we obtain [11]

$$\mid\psi\rangle = \sum_{j_t m_t} \sum_{\Delta h}^{N_c} \psi_{\Delta h}^{(j_t m_t)}(r)\, \frac{1}{r}\ \mid [Y_\Delta \Phi_h]_{j_t m_t}\rangle \ . \tag{A3}$$

In (A3) N_c denotes the total number of allowed Δ-hole states. The radial wave function is then given by the inversion of eq. (A3):

$$\psi_{\Delta h}^{(j_t m_t)}(r) = r\ ([Y_\Delta \Phi_h]_{j_t m_t} \mid \psi\rangle \ . \tag{A4}$$

The wave functions of the decay nucleon and the outgoing pion are also expanded in multipoles:

$$\mid \vec{q}_p^{\,\prime} \frac{1}{2}\, m_s'\rangle = 4\pi \sum_{l_p m_{l_p}} \sum_{j_p m_p} i^{l_p}\, \chi_{l_p}(q_p' r)\, (l_p\, m_{l_p}\ \tfrac{1}{2}\, m_s' \mid j_p\, m_p)\, Y^*_{l_p m_{l_p}}(\hat{q}_p')\ \Big[Y_{l_p}(\hat{r}) \otimes \chi_{\frac{1}{2}}\Big]_{j_p m_p} \ , \tag{A5}$$

$$\mid \vec{q}_\pi^{\,\prime}\rangle = 4\pi \sum_{l_\pi m_{l_\pi}} i^{l_\pi}\, \chi_{l_\pi}(q_\pi' r)\, Y^*_{l_\pi m_{l_\pi}}(\hat{q}_\pi')\, Y_{l_\pi m_{l_\pi}}(\hat{r}) \ , \tag{A6}$$

where χ_{l_p} and χ_{l_π} denote the distorted radial wave functions of the outgoing proton and pion, respectively. Using the expansion of eq. (A3) for the wave function $\mid\psi\rangle$ and using the expansions of eqs. (A5) and (A6) for the outgoing nucleon and pion wave functions we can rewrite the transition amplitudes for the quasi-free decay of the Δ as

$$T_{q.f.} = \sum_{j_t m_t} \sum_{\Delta h}^{N_c} \delta_{J_h^{'}, j_h} \delta_{M_h^{'}, m_h} \sum_{l_p m_{l_p}} \sum_{j_p m_p} \sum_{l_\pi j_\pi m_\pi} 2\sqrt{4\pi}^{\,3}\ \frac{f_{\pi N\Delta}}{m_\pi}\ i^{l_\Delta + l_\pi - l_p}\ \hat{\jmath}_\Delta\, \hat{l}_\Delta\, \hat{l}_\pi^2$$

$$Y_{l_p m_{l_p}}(\hat{q}_p') \, (-1)^{j_\pi - m_\pi} \, Y_{j_\pi m_\pi}(\hat{q}_\pi') \, (j_\Delta \, m_\Delta \, j_h \, m_h \mid j_t \, m_t) \, (j_\Delta \, m_\Delta \, j_\pi \, -m_\pi \mid j_p \, m_p) \left\{ \begin{array}{ccc} l_p & \frac{1}{2} & j_p \\ l_\Delta & \frac{3}{2} & j_\Delta \\ l_\pi & 1 & j_\pi \end{array} \right\}$$

$$(l_\Delta \, 0 \, l_\pi \, 0 \mid l_p \, 0) \, (l_\pi \, 0 \, 1 \, 0 \mid j_\pi \, 0) \, (l_p \, m_{l_p} \, \tfrac{1}{2} \, m_s' \mid j_p \, m_p) \, (1 \, \mu \, \tfrac{1}{2} \, \tau_p \mid \tfrac{3}{2} \, \tau_\Delta) \int dr \, (q_\pi' r) \, \chi_{l_p}(q_p' r) \, \chi_{l_\pi}(q_\pi' r) \, \psi^{(j_t \, m_t)}_{(\Delta \, h)}(r) \, . \tag{A7}$$

In eq. (A7) the Clebsch-Gordan coefficient $(1 \, \mu \, \frac{1}{2} \, \tau_p \mid \frac{3}{2} \, \tau_\Delta)$ describes the isospin coupling coefficient of the decay process and $\hat{x} = \sqrt{2x+1}$.

APPENDIX B: EXPLICIT FORMULAS FOR THE 2P EMISSION

Here we show the explicit formulas for the 2p emission processes. Due to the antisymmetrization of the two outgoing protons the transition amplitude for the 2p emission consists of the sum of the direct and exchange transition amplitudes. As a consequence of the arguments given in section II B we have to antisymmetrize only the π- and ρ-exchange interactions. The transition amplitude is given by (see Fig. 1):

$$T_{2N} = \langle J_h^{'} M_h^{'} ; \left[\vec{q}_{p^1}^{\,\prime} \frac{1}{2} \, m_{s^1}' , \vec{q}_{p^2}^{\,\prime} \frac{1}{2} \, m_{s^2}' \right]_{\mathcal{A}} \mid V_{pp,\Delta} \mid \psi \rangle \tag{B1}$$

$$= \frac{1}{\sqrt{2}} \left(\langle J_h^{'} M_h^{'} ; 1; 2 \mid V_\pi + V_\rho + V_\delta \mid \psi \rangle \; - \; \langle J_h^{'} M_h^{'} ; 2; 1 \mid V_\pi + V_\rho \mid \psi \rangle \right) \, . \tag{B2}$$

In eqs. (B1) and (B2) $J_h^{'}$ and $M_h^{'}$ represent the total angular momentum and the magnetic quantum number of the final (A-2)-nucleus; $\vec{q}_{p^1}^{\,\prime}$ and m_{s^1}' ($\vec{q}_{p^2}^{\,\prime}$, m_{s^2}') are the three-momentum and the spin projection of the outgoing proton 1 (2), respectively. Using the expansion of eq. (A3) for the wave function $\mid \psi \rangle$ and the multipole expansion of the nucleon wave functions of eq. (A5) we can write the matrix element for the direct decay graph in the following way:

$$\begin{aligned}
\langle J_h^{'} M_h^{'} ; 1; 2 \mid V_{pp,\Delta} \mid \psi \rangle = & \sum_{j_t m_t} \sum_{\Delta h}^{N_c} \sum_{l_{p^1} m_{l_{p^1}}} \sum_{j_{p^1} m_{p^1}} \sum_{l_{p^2} m_{l_{p^2}}} \sum_{j_{p^2} m_{p^2}} \sum_{j_{h^2}' \tau_{h^2}'} \sum_{J_p M_p} \sum_{J_1 J_2} \sum_{l_1 l_2} 2 \sqrt{6} \, (4\pi) \\
& i^{l_1 + l_2 - l_{p^1} - l_{p^2} + l_{h^2}' + l_\Delta} \, (-1)^{J_1 + J_2 - j_t} \, (-1)^{J_2 + M_2} \, \hat{J}_1^2 \, \hat{J}_2^2 \, \hat{J}_p \, \hat{J}_h' \, \hat{j}_{h^2}' \, \hat{j}_{p^1} \, \hat{j}_{p^2} \, \hat{j}_\Delta \, \hat{l}_1 \, \hat{l}_2 \, \hat{l}_{h^2}' \, \hat{l}_\Delta \\
& (j_{p^1} \, m_{p^1} \, j_{p^2} \, m_{p^2} \mid J_p \, M_p) \, (J_p \, M_p \, J_h' \, M_h' \mid j_t \, m_t) \, Y_{l_{p^1} m_{l_{p^1}}}(\hat{q}_{p^1}') \, Y_{l_{p^2} m_{l_{p^2}}}(\hat{q}_{p^2}') \\
& (l_{p^1} \, m_{l_{p^1}} \, \tfrac{1}{2} \, m_{s^1}' \mid j_{p^1} \, m_{p^1}) \, (l_{p^2} \, m_{l_{p^2}} \, \tfrac{1}{2} \, m_{s^2}' \mid j_{p^2} \, m_{p^2}) \, W(J_2 \, j_{p^1} \, j_t \, j_h \, ; \, j_\Delta \, J_1) \\
& (l_{h^2}' \, 0 \, l_2 \, 0 \mid l_{p^2} \, 0) \, (l_\Delta \, 0 \, l_1 \, 0 \mid l_{p^1} \, 0) \left\{ \begin{array}{ccc} j_{p^1} & j_{h^1}' & J_1 \\ j_{p^2} & j_{h^2}' & J_2 \\ J_p & J_h' & j_t \end{array} \right\} \left\{ \begin{array}{ccc} l_{p^1} & \frac{1}{2} & j_{p^1} \\ l_\Delta & \frac{3}{2} & j_\Delta \\ l_1 & 1 & J_2 \end{array} \right\} \left\{ \begin{array}{ccc} l_{p^2} & \frac{1}{2} & j_{p^2} \\ l_{h^2}' & \frac{1}{2} & j_{h^2}' \\ l_2 & 1 & J_2 \end{array} \right\} \\
& (-\sqrt{3}) \sum_{m=-1}^{1} (1 \, m \, \tfrac{1}{2} \, \tau_{p^1} \mid \tfrac{3}{2} \, \tau_\Delta) \, (1 \, m \, \tfrac{1}{2} \, \tau_{h^2}' \mid \tfrac{1}{2} \, \tau_{p^2}) \\
& \int dr_2 \, r_2^2 \, \phi_{n_{h^2}' \, (l_{h^2}' \, \frac{1}{2}) \, j_{h^2}'}(r_2) \, \chi_{l_{p^2}}(q_{p^2}' r_2) \int dr_1 \, r_1 \, v_{l_1, l_2, J_2}(r_1, r_2) \, \chi_{l_{p^1}}(q_{p^1}' r_1) \, \psi^{(j_t \, m_t)}_{(\Delta \, h)}(r_1) \, .
\end{aligned} \tag{B3}$$

Here $\phi_{n_{h^2}' \, (l_{h^2}' \, \frac{1}{2}) \, j_{h^2}'}$ denotes the radial hole wave function with quantum numbers $n_{h^2}' \, l_{h^2}' \, j_{h^2}'$. The non-local potential $v_{l_1, l_2, J_2}(r_1, r_2)$ is the sum of the central and tensor part of the decay interaction [32]:

$$v_{l_1,l_2,J_2}(r_1,r_2) = -4\pi\, \delta_{l_1,J_2}\, \delta_{l_2,J_2}\, V^C_{J_2}(r_1,r_2) \;+\; 4\pi\sqrt{6}\,(-1)^{J_2}\, \hat{l}_1\, \hat{l}_2\, (l_1\,0\,l_2\,0\,|\,2\,0)\, W(l_1\,1\,l_2\,1\,;\,J_2\,2)\, V^T_{l_1,l_2,J_2}(r_1,r_2)\;\; . \tag{B4}$$

For the direct matrix element $V^C_{J_2}$ and $V^T_{l_1,l_2,J_2}$ are the multipole expanded central and tensor parts of $V_\pi + V_\rho + V_\delta$ [32]; for the exchange matrix element they are the multipole expanded central and tensor parts of $V_\pi + V_\rho$.

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